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THE AMPLIFICATION OF STRESS IN FLEXIBLE STEEL ARCHES

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STRUCTURAL DIVISION

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THE AMPLIFICATION OF STRESS IN FLEXIBLE STEEL ARCHES[1]

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SYNOPSIS

The purpose of this paper is to present a simplified method that has been found useful in dealing with problems pertaining to the amplification of stress in flexible steel arches. Only fundamental concepts are used and extended to include the displacement of the arch axis. The increase, or amplification of the computed stress, at the quarter point of the loaded portion of the arch, when the displacements of the axis are included in the analysis, is developed and presented in chart form.

The amplification chart is useful in showing the entire range of structural action for curved members subjected to axial compression plus bending. The chart is divided into the following arbitrary ranges: the ordinary range, where the ordinary methods of structural mechanics apply with little error; the flexible range, where the ordinary methods of analysis must be extended to include the effects of displacement; and the dangerous range, where the amplification of stress is so large that there is danger of structural collapse.

The value of the chart is to show pictorially the overall range of structural action for a given arch, the magnitude of the amplification when displacements of the axis are included in the analysis, and what modifications may be made to reduce the amplification.

INTRODUCTION

The arch is one of the oldest structural forms and has become important in recent years because of the demand for long unobstructed spans. A typical steel bridge arch of the size and proportions herein discussed is shown in Figure 1. These proportions are similar to the Rainbow Bridge at Niagara.

Much time is spent in the preliminary design and analysis of long span steel arches because of the lack of a simplified method. It is not permissible to use the ordinary methods of structural mechanics for flexible arches, because the displacements which are normally neglected may increase or amplify the stresses and displacements. Refinements in analysis are for reasons of safety rather than for economy.

In order to determine the stress in an arch it is necessary to assume a design. In general, the effect of variations can only be determined by analyzing several designs. Because of the great amount of time and labor required, elaborate analyses are seldom practical.

A simplified method is herein presented in pictorial form by use of an Amplification Chart. This chart permits a study of the structural action for

1. A portion of a dissertation presented to the faculty of Yale University in candidacy for the degree of Doctor of Engineering, June 1951.
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a preliminary design, and the effects of variations in proportions are easily visualized. The chart may be modified for conditions other than originally specified; thus, one chart may serve for the different arch types, different computed stress, and for arches of different proportions and loading.

The simplified concepts used to develop the chart are explained and agree closely with the formal methods of structural mechanics when extended by successive correction to include the displacement of the arch axis.

Much literature is available on algebraic methods of analysis that include the effect of displacements of the arch axis.* The intent of this paper is not to discuss algebraic methods of analysis but rather to present a pictorial method which is adaptable to pre-design.

This paper has been limited to the presentation of a simplified method which has been found useful in the preliminary design of flexible steel arch ribs and does not deal with the details of design, the economics of design, the methods of proportioning arches, methods of fabrication and erection, or the interaction of deck and rib.

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*Some literature on algebraic methods of analysis for arches:

1. A.S.C.E. (American Society of Civil Engineers) Transactions, Vol. 110, Paper No. 2236. "Rainbow Arch Bridge Over Niagara Gorge," A Symposium, 1945.
2. Best, C. G. "Theory of Curved Beams." A.S.M.E. (American Society of Mechanical Engineers) Transactions, Vol. 68, pp. A 294-296, Dec. 1946.
3. Bresse, M. "Recherches analytiques sur la flexion et la resistance des pieces courbes." Paris, 1854.
4. Busch, B. "Die zentrische Knickung symetrisch belasteter gelenkloser Bogenträger mit veränderlichem Tragheitsmoment." Der Bauingenieur, Vol. 18, pp. 812-820, 1937.
5. Courbon, J. "Remarques sur l'application des formules de bresse au calcul des arcs." Annales des Ponts et Chaussees, Vol. 115, pp. 427-437, 1945.
6. Cross, H. "The Relation of Structural Mechanics to Structural Engineering," Fifth International Congress of Applied Mechanics, 1939.
7. Dauriac, P. A. "Flexion et flamboyement des arcs minces," La Houille Blanche,Vol. 2, No. 3, pp. 233-248, 1947.
8. Deutsch, E. "Das Knicken von Bogenträgern bei unsymetrischer Belastung," Der Bauingenieur, Vol. 21, pp. 353-360, Dec., 1940.
9. Dischinger, Fr. "Untersuchungen über die Knicksicherheit, die elastische Verformung und das Kriechen des Betons bei Bogenbrücken." Der Bauingenieur, Vol. 18, pp. 487-520, 538-552, 595-621; 1937.
10. Dischinger, Fr. "Elastische und plastische Verformungen der Eisentragwerke und insbesondere der Bogenbrücken," Der Bauingenieur, Vol. 20, pp. 53-63, 286-294, 426-437, 563-572; 1939.
11. Freudenthal, A. "Die Änderung des Spannungszustandes weitgespannter, flacher Eisenbetonbogen durch die plastische Dauerverformung des Betons," Beton u. Eisen, Vol. 34, pp. 176-184; 1945.

12. Freudenthal, A. "Deflection Theory for Arches," International Association for Bridge and Structural Engineering, Swiss Federal Institute of Technology, Vol. 3, p. 100; 1935.

13. Fritsche, J. "Knickbelastung der Bogenträger," Bautechnik, Vol. 3, p. 465; 1925

14. Fritz, B. "Theorie und Berechnung vollwandiger Bogenträger," Berlin 1934, Julius Springer, Inc.

15. Gaber, E. "Ueber die Knicksicherheit vollwandiger Bögen," Bautechnik, Vol. 12, pp. 646-651; 1934.

16. Hawranek, A. "Verformungstheorie des eingespannten Bogen mit einer Stützlinienachse," Der Bauingenieur, Vol. 18, pp. 719-727; 1937.

17. Hupner, "Application des series trigonometriques a l'etude du flambement des arches," Annales des Ponts and Chaussees, Vol. 105, No. 11, pp. 754-800; 1935.

18. Kasarnowsky, S. "Beitrag zur Theorie weitgespannter Brückenbögen mit Kampfergelenken," Der Stahlbau, Vol. 6, p. 61; 1931.

19. Kruck, G. "Die Methode der Grundkoordination," Zürich und Leipzig, Gebrüder Leeman und Co., 1937.

20. Levi, R. "Etude Generale du flambement des arches," Acadamie des Sciences - Comptes Rendus, Vol. 220, No. 13, pp. 436-438, 1945.

21. Levi, R. "Note sur le flambement et en particulier celui des arcs," Annales des Ponts et Chaussees, Vol. 117, pp. 345-366; 1947.

22. Linkler, E. "Formänderung und Festigkeit gekrümmter Körper, insbesondere der Ringe," Der Civilingenieur, Vol. 4, p. 232-246; 1858.

23. Melan, J. "Der biegsame eingespannte Bogen," Der Bauingenieur, Vol. 6, p. 143; 1925.

24. Müller-Breslau, H.F.B., "Theorie und Berechnung der eisernen Bogenbrücken," Baumgartner's Buchhandlung, Leipzig-Berlin, 1880.

25. Newmark, N. M. "Numerical Procedure for Computing Deflection, Moments, and Buckling Loads," A.S.C.E. (American Society of Civil Engineers) Transactions, Vol. 108, pp. 1168; 1943.

26. Ratzendorfer, J. "The Buckling of a Thin Circular Arch," Engineering, Vol. 150, pp. 284-285; 1940.

27. Wierzbricke, W., "Applications de la methode des moindras carres du calcul des arcs," Association International des Ponts and Charpentes, Vol. 5, pp. 263-274, 309-322, 333-342,375-422, 1937-38; Vol. 6, pp. 71-88, 265-276, 1938-1939.

Amplification Chart

The amplification chart, Figure 2, shows in pictorial form the per cent increase, or amplification, of the combined axial compression plus transverse bending unit stress when the displacements of the axis are included in the analysis. The chart is based on the combined stress at the quarter point of a three hinged arch with uniformly distributed live load over half the span. This loading does not produce maximum bending unit stress at the quarter point of the arch axis but was chosen for convenience.

The pressure line for unsymmetrical live load plus symmetrical dead load moves away from the arch axis. The displacements resulting from transverse bending are moment arms for the dead plus live load axial forces. Even though the displacements are small, the additional bending moments may be large because the live plus dead load axial forces are large. In flexible members the bending stresses and displacements are increased or amplified by the additional bending moment resulting from the product of the large axial forces and the transverse bending displacements. The total combined axial-compression plus amplified bending stress may be represented by the following beam formula modified for additional bending. This formula has been completely derived in a following chapter.

$$f_t = f_a + f_b \left(1 / 1 - \frac{f_a}{nE} \left(\frac{L}{r}\right)^2 \right)$$

where f_a is the total dead plus live load axial stress

f_b is the dead plus live load bending stress before amplification

$\frac{1}{1 - \frac{f_a}{mE}\left(\frac{L}{r}\right)^2}$ is the amplification factor.

n is a shape factor that depends on the loading and properties of the member. For the chart of Figure 2, n = 1/10.

$\frac{L}{r}$ is the slenderness ratio.

E is the Modulus of Elasticity. For Figure 2, E = 30,000,000 psi

The total stress computed in the usual way, neglecting the effect of the displacements, may be represented by the following formula:

$$f = f_a + f_b$$

The equation for the total combined axial plus amplified bending stress can be developed for the straight beam and then extended to include curved members and the different arch types. A discussion of the magnitude of the errors that result from simplification when displacements are considered is presented in the derivation of the basic equation below.

The amplification chart is based on the formula for the total combined axial plus amplified bending stress. The chart shows the amplification of stress f_t/f plotted against the slenderness ratio L/r and the ratio of axial and bending stress f_a/f_b or f_b/f_a. By use of both f_a/f_b amd f_b/f_a, extremes may be conveniently shown on one chart.

In addition to showing the magnitude of the amplification of stress when displacements are included in the analysis, the chart also shows arbitary

ranges of structural action.

Use of The Amplification Chart

The amplification chart may be used to determine the magnitude of the amplification of stress when displacements of the axis are included in the analysis, to show the influence of modifications on the amplification of stress, and to describe in a general way the structural behavior of beam columns and arches in the entire range of structural action.

To use the chart to determine the magnitude of the amplification of the bending stress at the quarter point of an arch, the ratios needed to enter the chart are computed:

First, the slenderness ratio L/r is computed and then modified for the computed stress and type of arch. Here L is one-half of the span length and r is the radius of gyration at the quarter point of the arch rib. The chart shown in Figure 2 is based on a computed stress of 20,000 pounds per square inch. If the computed stress, before amplification, $f = f_a + f_b$, is other than 20,000 pounds per square inch, the slenderness ratio L/r, needed to enter the chart, is determined by multiplying the slenderness ratio for the stress other than 20,000 psi by the square root of the quotient of the other stress divided by 20,000 psi.

If the arch is three-hinged, no further modification in the slenderness ratio is necessary. If the arch is of different type, it is necessary to further modify the slenderness ratio by the following approximate values which may be determined approximately by sketching the pressure line for the loads on the different arch types:

Type of Arch	Multiply L/r by the following
Three-Hinged Arch	1.0
Two-Hinged Arch	0.9
One-Hinged Arch	0.8
Hingeless Arch	0.7

The above values are approximations, determined by rounding off computed values. It is important to note that the above values of $\frac{L}{r}$ pertain to the bending stress at the quarter point of the portion of the arch subjected to live load. These values vary with the loading, shape of arch axis, position along the arch axis, and properties of the cross section.

Second, the ratio of axial to bending stress f_a/f_b, or the ratio of f_b/f_a is computed, whichever is smaller. These two ratios are all that are needed to enter the amplification chart. Variations in rise of arch, depth of rib, moment of inertia, area of section are included when the ratio of axial and bending stress is determined.

The chart shown in Figure 2 is entered at the bottom with the ratio of f_a/f_b or f_b/f_a, whichever is smaller, and thence vertically up to the value of L/r modified for the computed stress and type of arch. The intersection shows the range of structural action and the magnitude of the amplification of the total stress of the quarter point when displacements of the axis are included in the analysis.

It is also possible to express the ratio of axial and bending stress in terms of the properties of the arch and the loading. Thus, a similar chart could be constructed where the abscissa are expressed in terms of the ratio of the

rise of arch to the depth of rib for a given form of cross section and the ratio of the dead to live load at the quarter point of the arch for a given radius of gyration.

It may be of interest to note that the amplification of stress is little influenced for smaller values of L/r by variations in the ratio of f_a/f_b and f_b/f_a either side of $f_a = f_b$.

EXAMPLE in the use of the chart to determine the magnitude of the amplification of stress: Consider any steel arch of proportions similar to Figure 1. From the proportions of the arch rib and the loading, the ratios needed to enter the chart are computed.

First, the slenderness ratio L/r is computed, and then modified for the computed stress and type of arch. Here L is one-half of the span length and equal to 950/2 or 475 feet, and r is the radius of gyration at the quarter point and equal to $\sqrt{I/A} = \sqrt{12{,}3000 \text{ in.}^2 \text{ ft.}^2 / 554 \text{ in.}^2} = 4.76$ ft.

Therefore, L/r = 475 ft. / 4.76 ft. = 100. Since the chart is based on a computed stress of 20,000 psi, it is necessary to modify the slenderness ratio for any other computed stress. The computed stress is 19,100 psi.; therefore, the slenderness ratio would be modified by the following:

$$L/r)_{\substack{\text{to enter} \\ \text{chart}}} = L/r \sqrt{\text{other stress}/20{,}000}$$

$$= 100 \sqrt{19{,}100/20{,}000} = 97.5.$$

The Rainbow Arch is hingeless; thus, the slenderness ratio is further modified by 0.70. The slenderness ratio needed to enter the chart is 97.5 x 0.7 = 68.

Second, the ratio of the bending stress to the axial stress is computed:

$$f_b/f_a = 7.38 / 11.72 = 0.63$$

The chart shown in Figure 2 is entered at the bottom with the ratio of $f_b/f_a = 0.63$; thence, vertically up to the value of L/r = 68 which has been modified for the computed stress and type of arch. The intersection shows the range of structural action and the approximate value of the amplification of the stress when changes in the dimensions of the arch axis are included. The amplification of the stress at the quarter point interpolated from the chart is about 8 per cent. It may be of interest to note that the designers of the Rainbow Bridge computed an amplification of stress of 7 per cent.[3]

Use of The Amplification Chart to Show The Influence of Modifications on The Amplification of Stress

The amplification of stress in arches subjected to combined axial compression and bending is influenced by the slenderness ratio L/r, the type of arch, the ratio f_a/f_b and the computed stress $f = f_a + f_b$.

The amplification of stress may be reduced by decreasing the slenderness ratio. For a given length between points of inflection the amplification of stress decreases for increasing values of the radius of gyration.

3. This value was obtained from Paper No. 2236, A.S.C.E. Transactions, Volume 110 (1945) page 33.

A higher allowable computed stress is permitted for certain types of steel and for wind load. The relative amplification of stress is influenced by variations in the allowable or computed stress $f = f_a + f_b$. An increase in the computed stress results in a disproportionate increase in amplification. The amplification chart is useful in showing the influence of changes in the computed stress. For example, consider an arch with slenderness ratio of 100 and a computed stress before amplification of 20,000 psi. If the total stress is increased 50%, from 20,000 psi to 30,000 psi, the slenderness ratio is modified by:

$$\sqrt{\text{other stress}/20{,}000} = \sqrt{30{,}000/20{,}000} = \sqrt{1.5} = 1.23$$

The corresponding amplification is increased from 25% for L/r = 100, to 45% for L/r = 123: a net increase of 80%.

Figure 3 shows the amplification of the total stress divided by the allowable stress, f_t/f, plotted against the slenderness ratio, L/r, for different allowable unit stresses. This chart is for a flexible arch rib of silicon steel with a yield point stress of 45,000 psi.

For larger values of L/r the amplification of the total stress becomes disproportionately larger than the corresponding increase in allowable unit stress, f.

The amplification of stress is little influenced for values of L/r less than 80 by changes in f_a/f_b when this ratio is nearly unity. For decrease in relatively small values of f_a/f_b and f_b/f_a the amplification of stress is decreased. This is true because there is little bending to amplify in the range of all axial stress, and small amplification in the range of all bending stress because the axial stress is small.

Use of Chart to Describe Structural Action

The amplification chart may be useful in describing the range of structural action of arches. The charts have been divided into three arbitrary ranges: the ordinary range, the flexible range, and the dangerous range.

The ordinary range shows where the analysis by customary methods of structural mechanics give sufficient results. In the ordinary range, changes in the dimensions of the arch axis and changes in displacement of the arch rib have little influence on the amplification of stress; therefore, to that extent, superposition of effects is applicable. In the ordinary range, for a given slenderness ratio less than 70, and for values of f_a/f_b from 0.5 to 2.0, the amplification of stress remains practically constant, irrespective of loading and proportions of the arch. For decreasing values of f_a/f_b and f_b/f_a less than 0.5 the amplification of stress decreases for constant L/r. The chart in Figure 2 has been constructed for a total stress before amplification of 20,000 psi. If the computed stress is greater than 20,000 psi., it is necessary to consider the corresponding disproportionate increase in the amplification. In the ordinary range elastic buckling is highly improbable except for relatively small values of f_b/f_a where the amplification of displacements is relatively large.

In the flexible range, the ordinary methods of structural mechanics must be modified and extended to include the effects of displacements, superposition of effects is not applicable, and it is not permissible to separate live

and dead load. The amplification of stress in this range has been arbitrarily limited from 10% to 50%. The area of constant amplification of stress for a given L/r is less pronounced in the flexible range than in the ordinary range. The amplification of stress increases for increasing values of f_a/f_b up to a certain point and then decreases. Variations in loading and properties of the arch have more influence on the amplification of stress in the flexible range than in the ordinary range. Amplification of stress is more pronounced for small changes in the slenderness ratio. Attention is given to variations in the computed stress since increases in the computed stress result in disproportionate increases in the amplification. Elastic buckling may be possible for the relatively larger values of L/r and for the values of f_a/f_b greater than one.

In the dangerous range the amplification of stress is greater than 50%. In this range the amplification of stress and displacement is so large that there is danger of elastic or plastic buckling. Small changes in slenderness ratio are associated with disproportionate changes in amplification. Variations in loading and properties of the arch are relatively more important in the dangerous range. For a constant L/r, increasing the ratio of f_a/f_b will end in elastic or plastic buckling. The line separating elastic and plastic buckling depends to some extent on the type of steel. The higher allowable stresses associated with silicon and nickel steel result in disproportionate increases in amplification.

The chart is of value to show immediately, for an assumed arch, if danger exists, and what modifications are indicated to get out of danger without making elaborate computations.

Amplification

The bending moment obtained from the pressure line for the live load alone is not the actual moment in the structure. The bending resulting from vertical loads is increased or amplified by additional bending due to the axial forces. Normally an arch is designed so that the pressure line for the loads fits the arch axis closely. The bending moment is computed by multiplying the thrust by the distance from the pressure line to the arch axis. If the displacements of the arch axis are neglected, the same bending moment will be determined for the live load pressure line as for the dead load plus live load pressure line. Thus by the usual theory, it is permissible to separate the live from the dead load and add partial effects to get total effects.

In flexible members the displacements of the arch axis, even though small, may be a large part of the dead plus live load pressure line as shown in Figure 4. If the displacements of the arch axis are included, larger bending moments will be determined for the live plus dead load pressure line than for the live load pressure line. The transverse displacements are moment arms for the combined live plus dead load axial forces. When the displacements of the axis are included, it is not permissible to separate the live from the dead load. Not only are the bending moments and displacements increased or amplified by the live plus dead load axial forces, but also the sums of partial effects of live and dead load are not equal to toal effects. However, it is often convenient to set an arbitrary allowable increase in the results obtained by including the effects of the displacements over the results obtained by assuming that the structure does not deform, and permit superposition within this range.

The stages of additional bending continue and may be represented by a

series. This series may or may not converge. If the bending continues to increase (series does not converge), the structure will collapse.

Consider a straight member subjected to both axial and transverse loads as shown in Figure 5. Bending moment diagrams for beams subjected to different loading conditions are shown in Figure 6. The equation for the displacement at the center of the beam is $\Delta = n\,ML^2/EI$, where M is the ordinate of the bending moment diagram at the center of the beam, and n is the shape factor that may vary depending on the loading and the proportions of the structure.

The M or M/EI diagrams are grouped in Figure 7. The value of n may be determined, by interpolation, for any other loading by superimposing the particular M or M/EI diagram in question on Figure 7. The bending moment for the transverse loads is increased by additional bending due to the axial forces. The shape factor for the transverse loads n_b is not the same as for the axial loads n_a.

The additional bending moment, Figure 5, is $P_a \Delta b$ which causes further displacement

$$n_a' P_a \Delta_b L^2/EI \text{ and further bending moment}$$

$$P_a n_a' P_a \Delta_b L^2/EI.$$

This process is continued and the total bending moment may be represented by the following series:

$$M = M_b + P_a \Delta_b + P_a \Delta_b n_a' (P_a L^2/EI)$$

$$+ P_a \Delta_b n_a' n_a'' (P_a L^2/EI)^2 + \ldots$$

This may be written as:

$$M = M_b \cdot \frac{1 - n_a P_a L^2/EI + n_b P_a L^2/EI}{1 - n_a P_a L^2/EI}$$

By setting $n_a = n_b$ the following simplified expression is obtained:

$$M = M_b / 1 - n_a P_a L^2/EI.$$

This equation is approximate. Slight errors exist by neglecting successive changes in n_a. A scale on the magnitude of the error in bending moment, as a result of setting $n_a = n_b$, may be obtained from Figure 8. The corresponding error in the combined axial plus bending stress will be less.

The quantity $1/1 - n_a P_a L^2/EI$ is the amplification factor. Since $r^2 = I/A$,

and $f_a = P_a/A$, and by defining f_x as $E/n_a(L/r)^2$, the amplification factor may also be written as $1/1-f_a/f_x$.

The total stress for the straight member subjected to both axial and transverse loads may be computed by the following beam formula modified for additional bending: the effect of temperature, rib shortening, movement of abutment or hinge in a straight member is not included.

$$f_t = f_a + f_b\,(1/1-f_a/f_x)$$

The previous discussion and amplification formula for a horizontal straight member also holds approximately for a curved member with supports on the same elevation. Consider a curved member (circular for convenience) of rise h with a concentrated load at the center of the beam as shown in Figure 9a. If small changes in span length are neglected, the bending moment for the curved beam is the same as for the corresponding horizontal straight beam. The displacement at the crown is slightly larger for a curved member than for a straight member. Consider a circular member with constant moment of inertia and various ratios of rise of arch to span length h/L. The ratio of the displacement at the center of a curved member, with concentrated load at the center, to a similar supported and loaded straight beam is shown in Figure 9b. For arches of ordinary proportions the increase in the displacement of a curved member over the straight member is small and in most cases may be neglected. However, if it seems desirable to refine computations to include the effect of curvature, the slenderness ratio may be increased by the small percentage shown in Figure 9b.

The displacement at the center of a beam is influenced by the difference in elevation of the end supports and by the shape of the beam. Consider a straight beam, an inclined beam, and an inclined curved beam as shown in Figure 10a. The displacement at the center of a straight beam with supports on different elevations is greater than the corresponding straight beam. The increase in displacement of the inclined beam over the horizontal beam is shown in Figure 10b. Values for an inclined curved beam are also shown.

Thus, the amplification of stress in flexible steel arch ribs may be determined approximately by the following beam formula modified for additional bending: $f_t = f_a + f_b\,(1/1-f_a/f_x)$

An estimate of the error introduced by neglecting the curvature of the axis and relative difference in elevation between points of inflection may be determined from Figure 10b. For flexible steel arch ribs of ordinary proportions this error is usually small and may be neglected.

SUMMARY

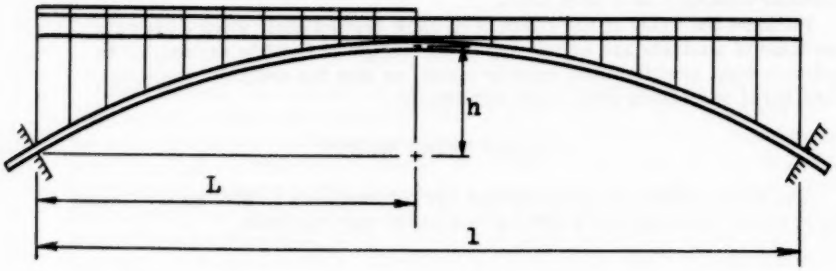
This paper has been limited to the development and presentation of an amplification chart which the writer has found useful in the pre-design of flexible steel arches. The chart shows the overall picture of the increase or amplification of stress at the quarter point of an arch when displacements of the axis are included in the analysis. In the development of the chart only familiar concepts have been used.

Perhaps the value of the chart is to show immediately when elaborate methods of analysis are necessary in the design of flexible arches, or to indicate what modifications may be made, so that the ordinary methods of structural mechanics give sufficient results.

ACKNOWLEDGEMENTS

The writer wishes to acknowledge his indebtedness to Mr. Hardy Cross, from whose teaching and guidance this paper was inspired.

LIVE LOAD = 1.4 KIPS/FT. DEAD LOAD = 9.0 KIPS/FT.



PROPORTIONS:

SPAN LENGTH	l	= 950 FT.
RISE OF ARCH	h	= 150 FT.
DEPTH OF ARCH RIB	d	= 12 FT.
SECTION MODULUS	I/c	= 24,600 $IN.^3$
RADIUS OF GYRATION	r	= 4.76 FT.
MOMENT OF INERTIA	I	= 12,300 $IN.^2$ $FT.^2$
AREA OF CROSS SECTION	A	= 554 IN^2
$\frac{\text{SPAN LENGTH}}{\text{RISE OF ARCH}}$	$\frac{l}{h}$	= 6.3
MODULUS OF ELASTICITY	E	= 30 x 10^6 PSI.

FIG. 1. TYPICAL STEEL BRIDGE ARCH WITH PROPORTIONS.

CHART SHOWING THE AMPLIFICATION OF STRESS, IN PERCENT, AT THE QUARTER POINT OF A THREE-HINGED ARCH FOR VALUES OF THE SLENDERNESS RATIO L/r AND THE RATIO OF AXIAL AND BENDING STRESS.

THIS CHART IS FOR A THREE-HINGED ARCH WITH A STRESS, BEFORE AMPLIFICATION, OF 20,000 POUNDS PER SQUARE INCH AT THE QUARTER POINT. FOR ANY OTHER STRESS MULTIPLY L/R BY $\sqrt{\text{OTHER STRESS}/20{,}000 \text{ PSI}}$

FOR THE ARCH TYPES MULTIPLY L/r BY THE FOLLOWING APPROXIMATE VALUES: L IS ONE HALF THE TOTAL SPAN LENGTH.

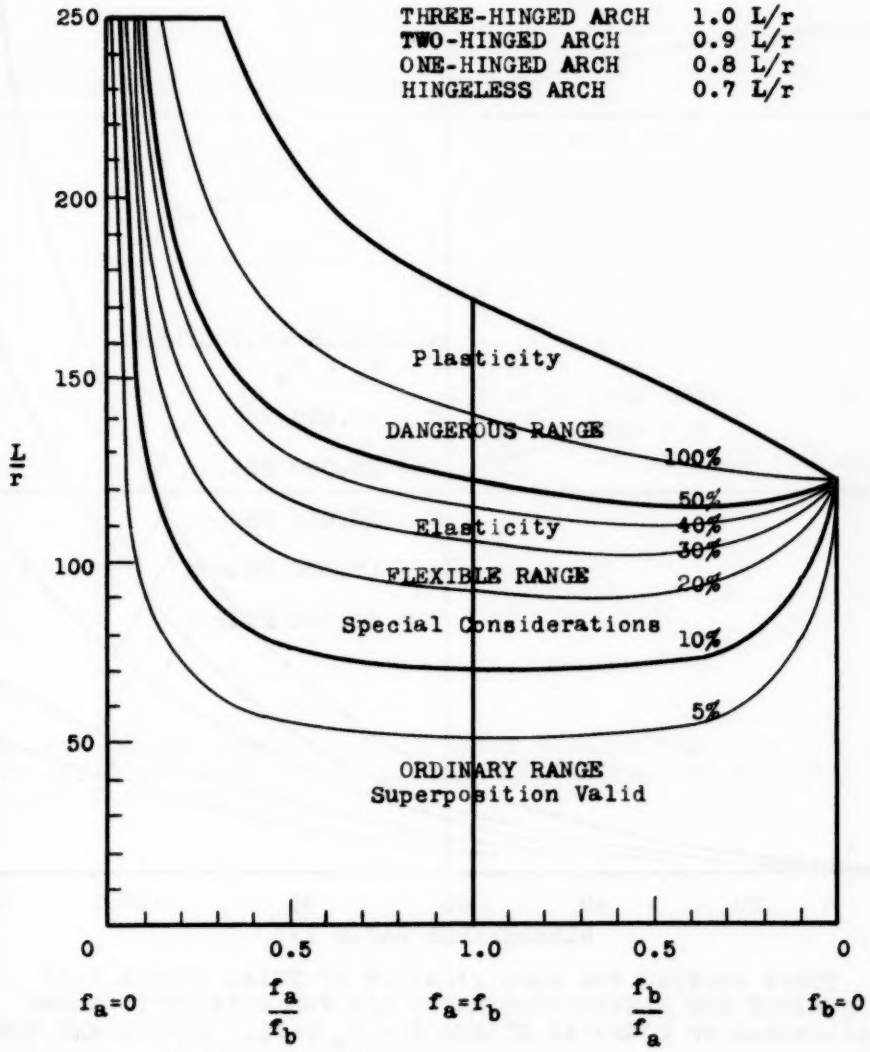


FIG. 2. AMPLIFICATION CHART

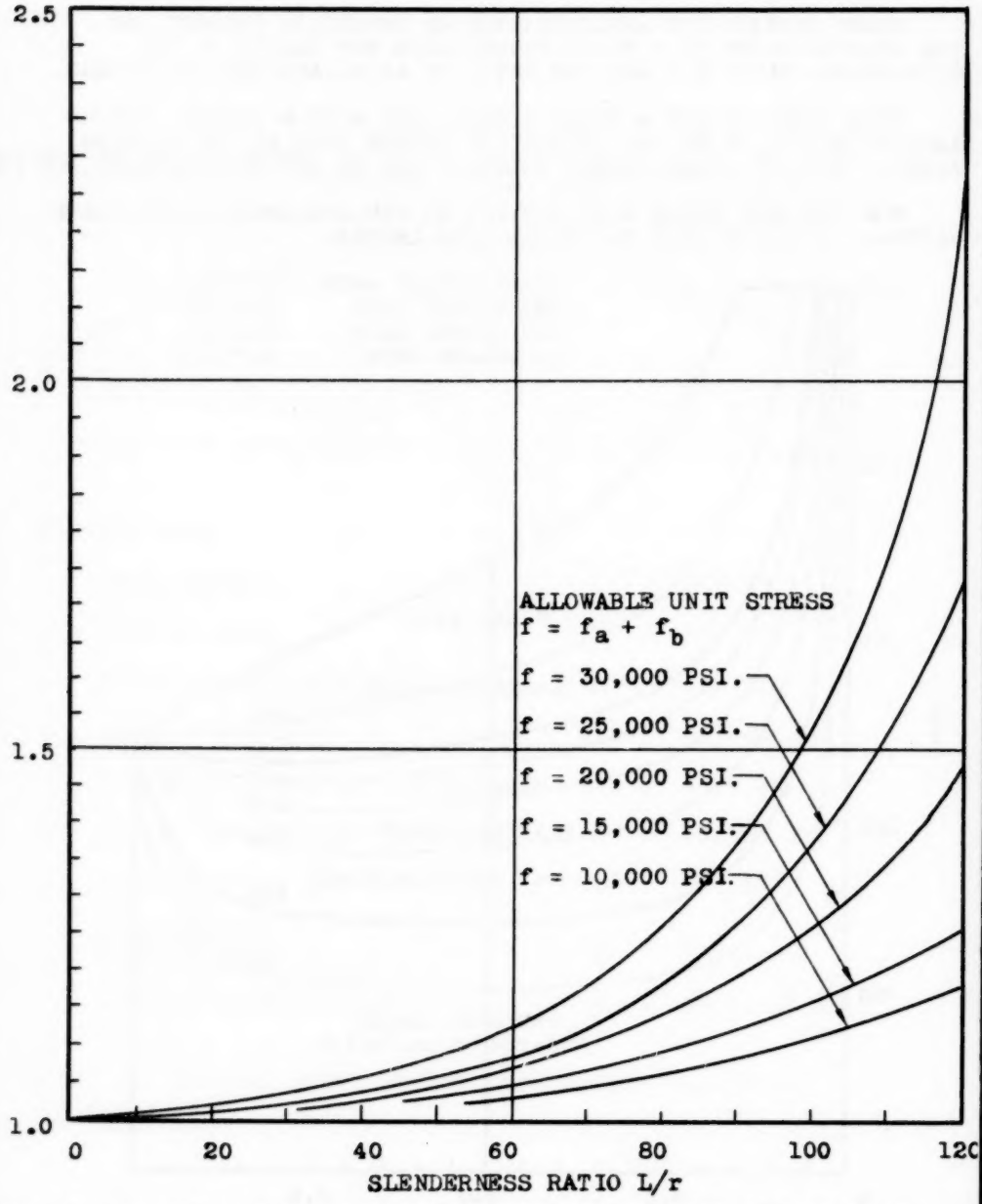


FIG. 3. CURVE SHOWING THE AMPLIFICATION OF TOTAL STRESS f_t/f PLOTTED AGAINST THE SLENDERNESS RATIO L/r FOR DIFFERENT VALUES OF THE ALLOWABLE OR COMPUTED STRESS $f = f_a + f_b$. CURVES ARE FOR $f_a = f_b$.

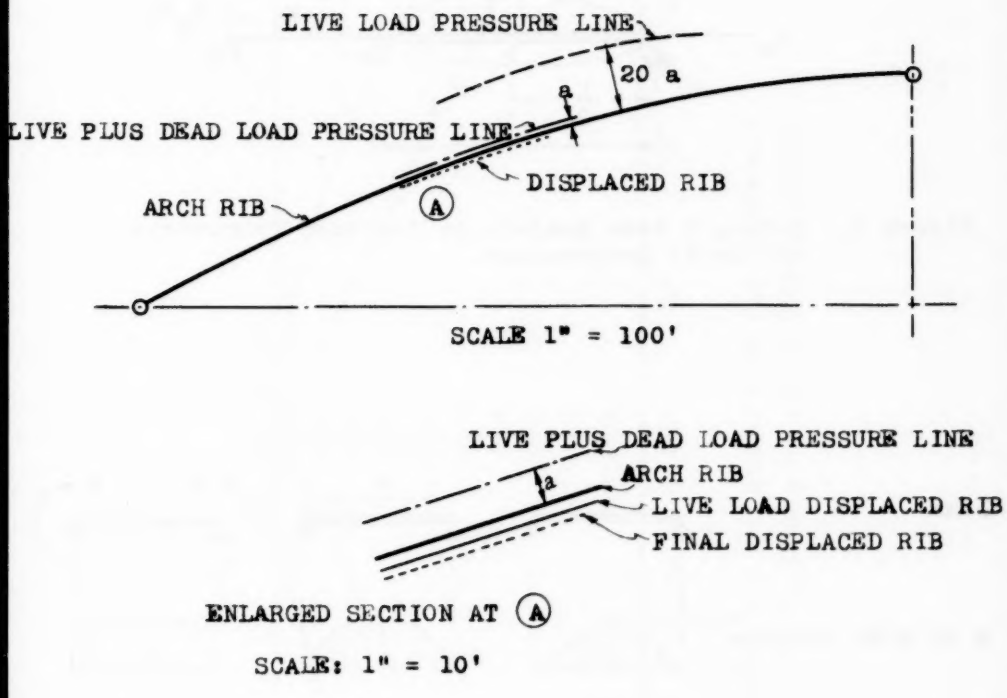


FIG. 4. SKETCH SHOWING THE RELATION BETWEEN THE LIVE LOAD AND LIVE PLUS DEAD LOAD PRESSURE LINE WITH RESPECT TO THE ARCH AXIS AND THE DISPLACED RIB.

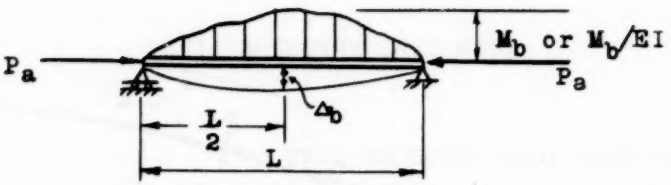


Figure 5. Straight Beam Subject to Combined Transverse and Axial Compression.

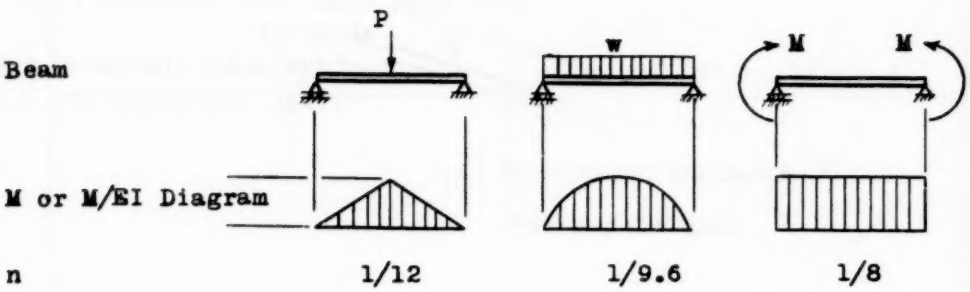


Figure 6. Bending Moment Diagram For Beam of Length L With Shape Factor n.

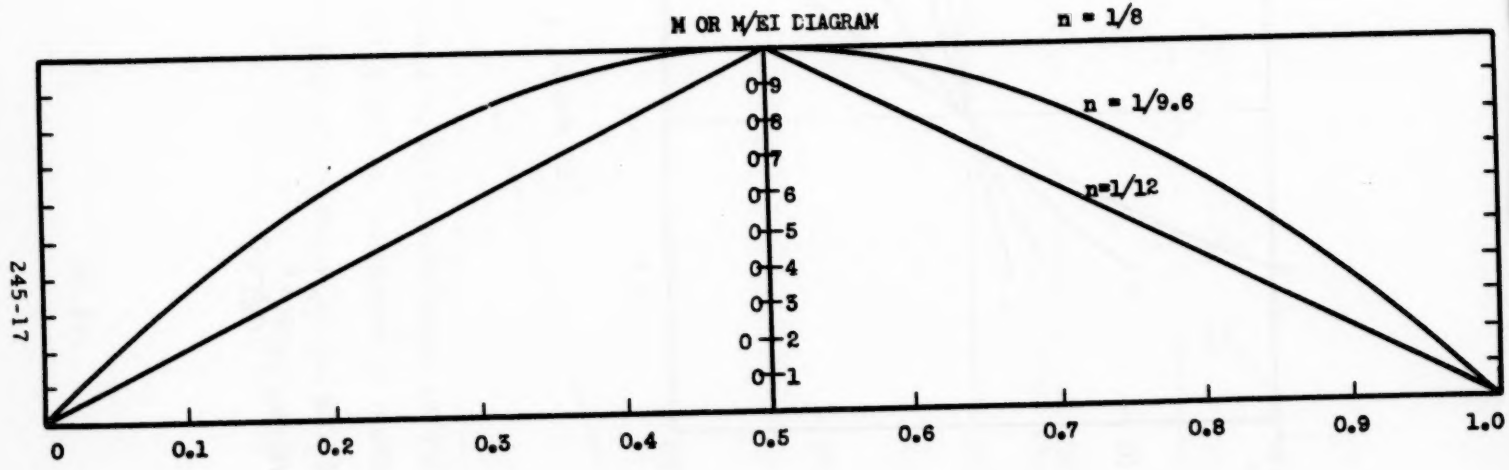


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FIGURE 7. M OR M/EI DIAGRAM TO BE USED TO DETERMINE VALUES OF n FOR VARIOUS TYPES OF LOADING

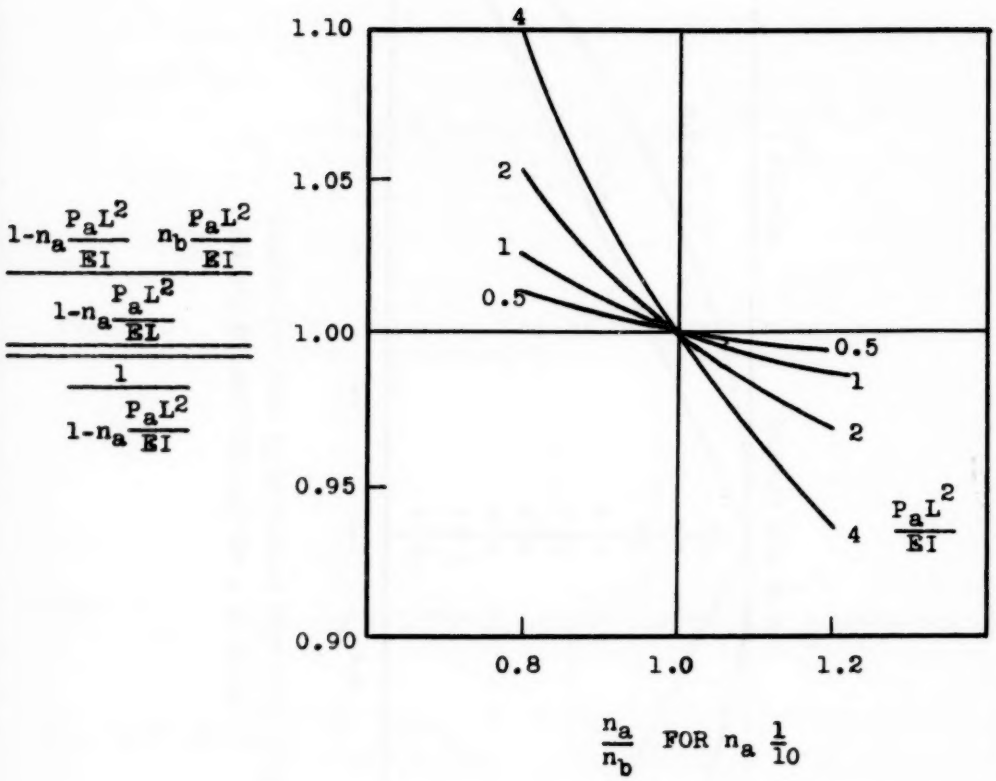

FIGURE 8. CURVE SHOWING ERROR RESULTING FROM SETTING n_a EQUAL TO n_b. THE ERROR IS EXPRESSED AS THE RATIO OF THE ACTUAL VALUE TO THE SIMPLIFIED VALUE WHICH IS PLOTTED AGAINST n_a/n_b FOR VALUES OF $\frac{P_aL^2}{EI}$.

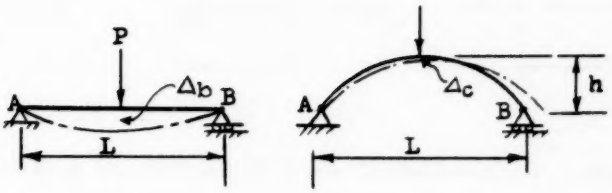

FIGURE 9a. DISPLACEMENT AT THE CENTER OF A STRAIGHT AND CURVED BEAM

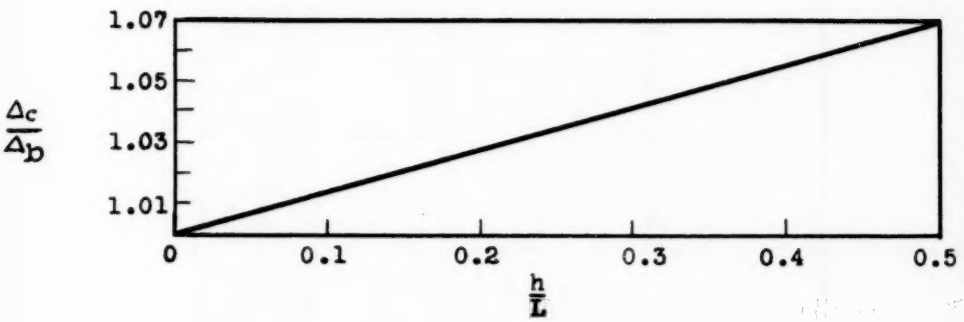

FIGURE 9b. CURVE SHOWING THE RATIO OF THE DISPLACEMENT AT THE CENTER OF A CURVED TO STRAIGHT BEAM SUBJECTED TO A CONCENTRATED LOAD AT THE CENTER.

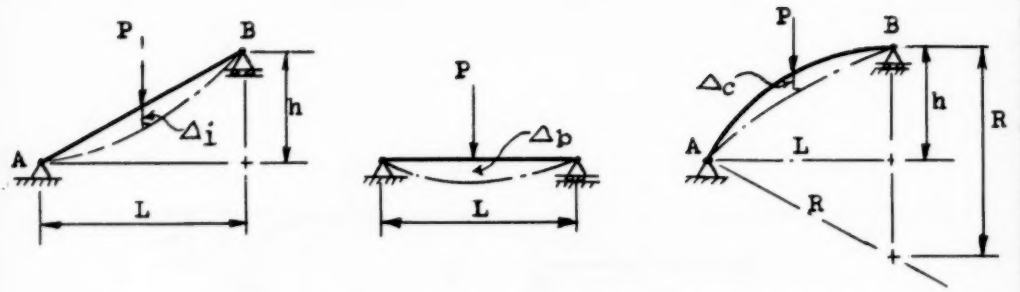

FIGURE 10a. DISPLACEMENT AT THE CENTER OF A HORIZONTAL STRAIGHT, INCLINED STRAIGHT, AND INCLINED DURVED BEAM.

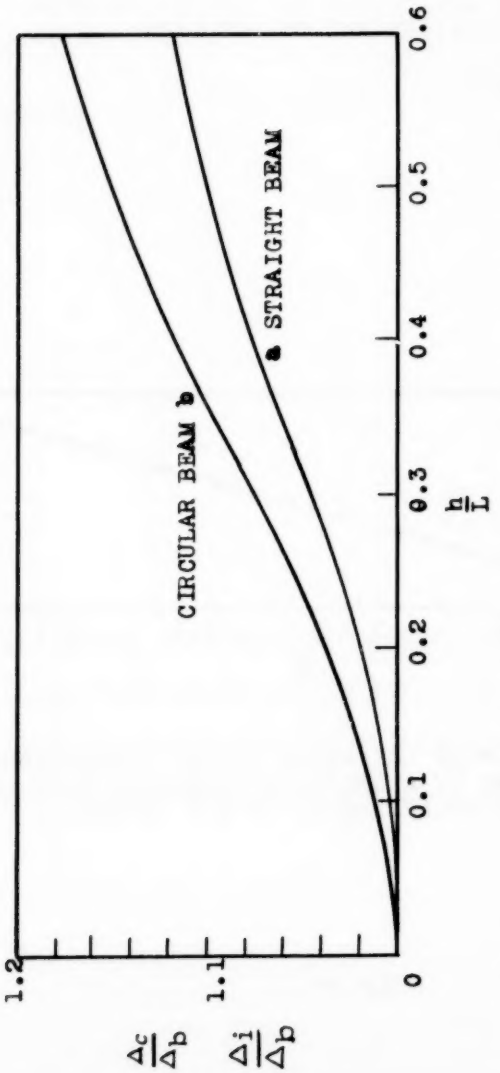

FIGURE 10b. CURVE a SHOWING THE RATIO OF THE DISPLACEMENT AT THE CENTER OF A BEAM SUPPORTED ON THE SAME ELEVATION TO THE DISPLACEMENT AT THE CENTER OF A BEAM WITH SUPPORTS ON DIFFERENT ELEVATIONS SUBJECTED TO A CONCENTRATED LOAD AT THE CENTER. CURVE b IS FOR A CIRCULAR BEAM WITH DIFFERENT RADII TANGENT TO THE HORIZONTAL AT B AS SHOWN IN FIGURE 10a.

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